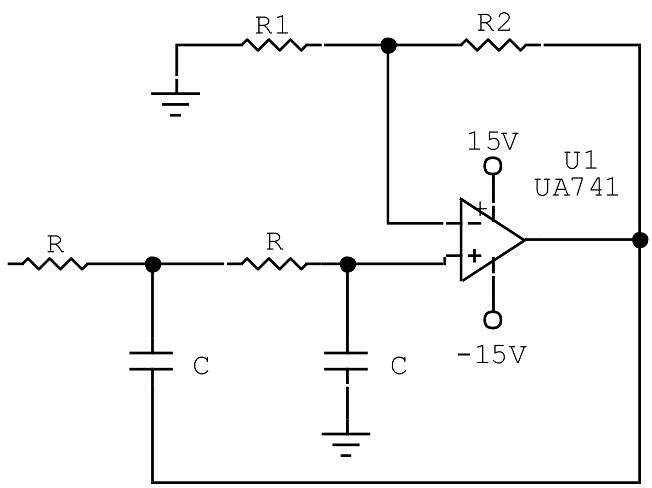
Part A



2nd Order low pass filter

1.

The specifications state that this filter has a 3dB frequency of 10kHz

$$f_{3dB} = 10kHz$$

$$\omega_{3dB} = 62.832krad/s$$

This cut off frequency must also satisfy the following equation

$$\omega_{3dB} = \frac{1}{RC}$$

And $R = 10k\Omega$, therefore C must be

$$62.832k = \frac{1}{10kC}$$

$$C = 1.59nF$$

The transfer function was given as

$$H(s) = A_M \frac{1/(RC)^2}{s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}}$$

Where the denominator follows the following format

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

And ω_n was already seen to be $\frac{1}{RC}$, so then ζ must be

$$\zeta = \frac{3 - \mathrm{A_M}}{2}$$

And solving for A_M yields

$$A_M = 3 - 2\zeta$$

The damping factor ζ lies somewhere between 0 and 2. The filter is critically damped when

$$\zeta = \frac{1}{\sqrt{2}}$$

So the above equation becomes

$$A_M = 3 - \frac{2}{\sqrt{2}}$$

$$A_M = 1.59$$

Now there are two equations and two unknowns so R_1, R_2 can be solved for

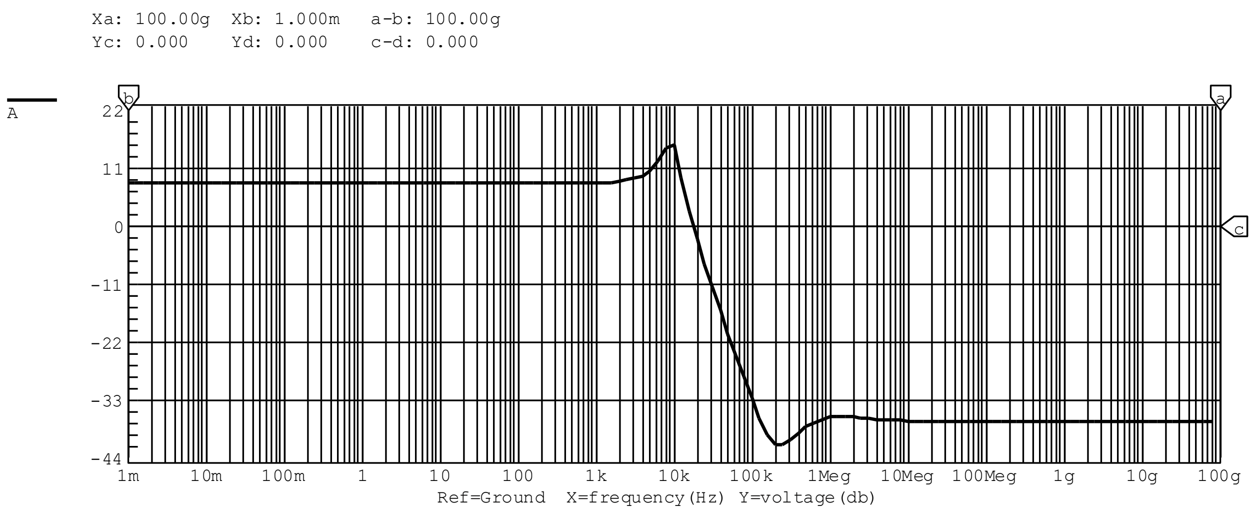
$$R_1 + R_2 = 10k$$

$$1.59 = 1 + \frac{R_2}{R_1}$$

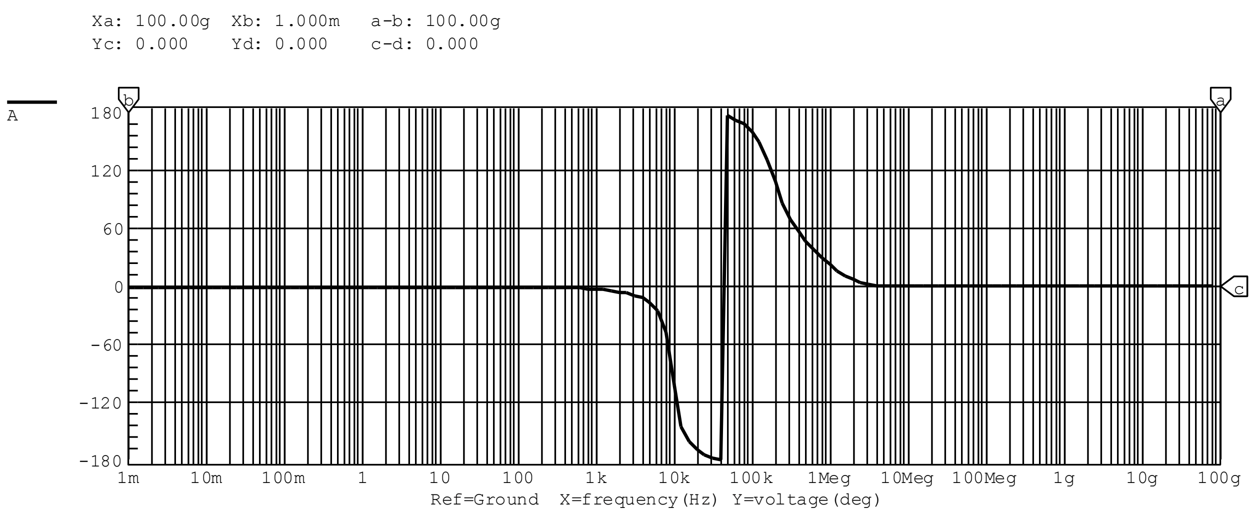
$$R_1 = 3.694k\Omega$$

$$R_2 = 6.306k\Omega$$

Bode Plots:

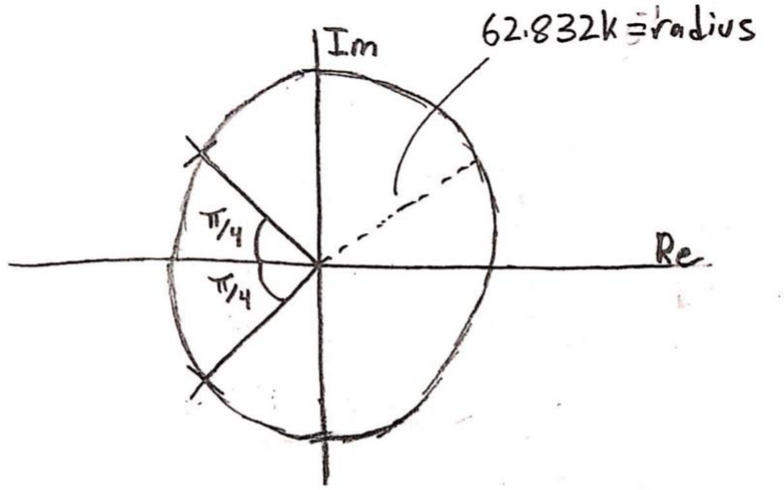


Magnitude Plot



Phase Plot

S Plane Plot (Pole Locations):



2.

The transfer function was given as

$$H(s) = A_M \frac{1/(RC)^2}{s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}}$$

If $R_2 = 6.66k$ and $R_1 = 3.33k$, both these equations are till satisfied and now $A_M = 3$

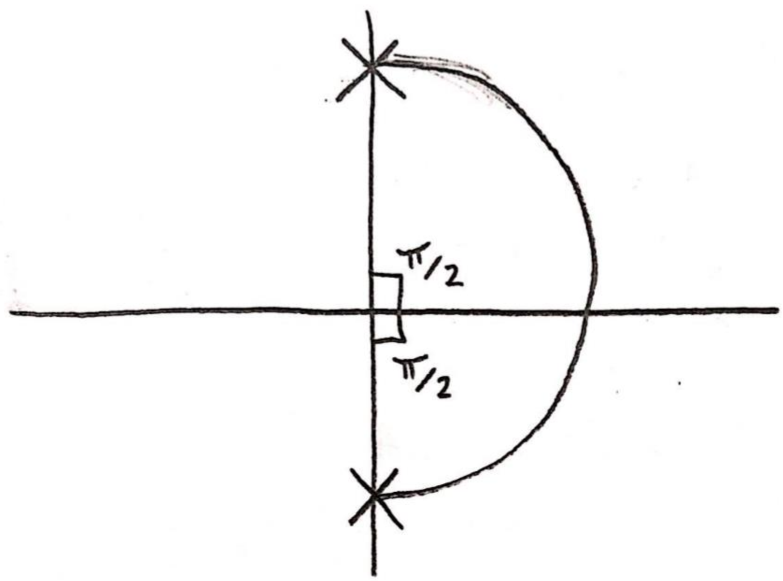
$$R_1 + R_2 = 10k$$

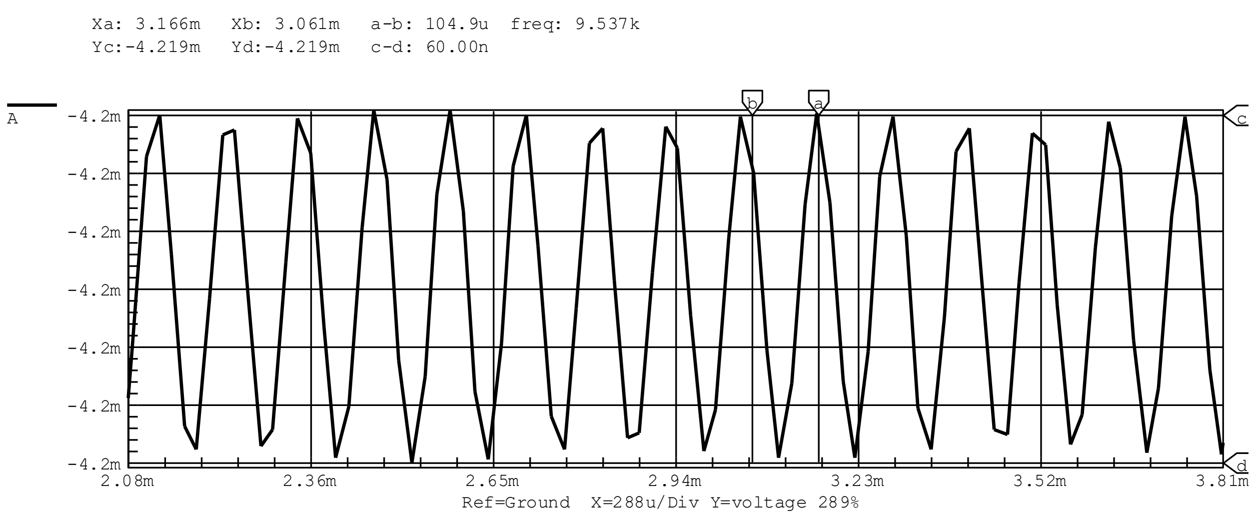
$$A_M = 1 + \frac{R_2}{R_1}$$

$$A_M = 3$$

At this point, the middle term of the denominator is 0 and the poles lie entirely on the imaginary axis

S Plane Plot (Pole Locations):



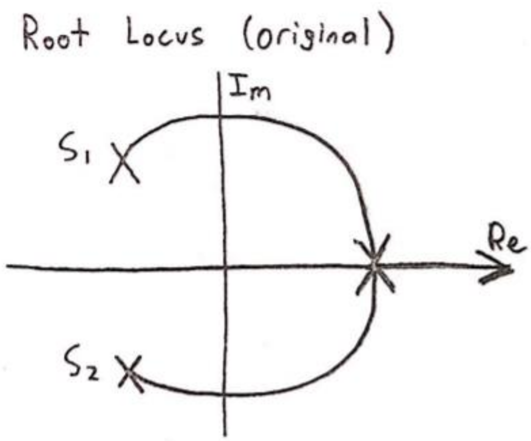


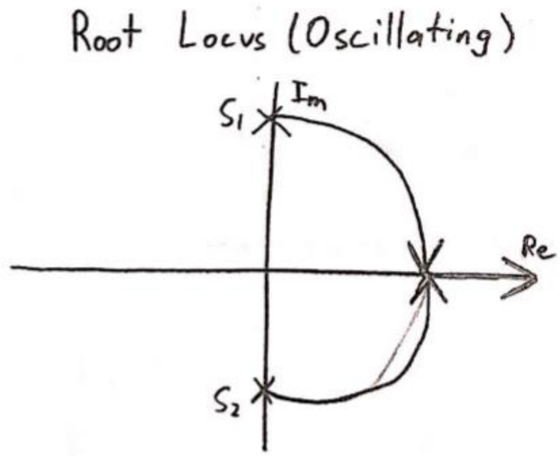
Oscillating output with grounded input

From this plot, the oscillating frequency can be obtained

$$f = 9.537kHz$$

Root Locus:

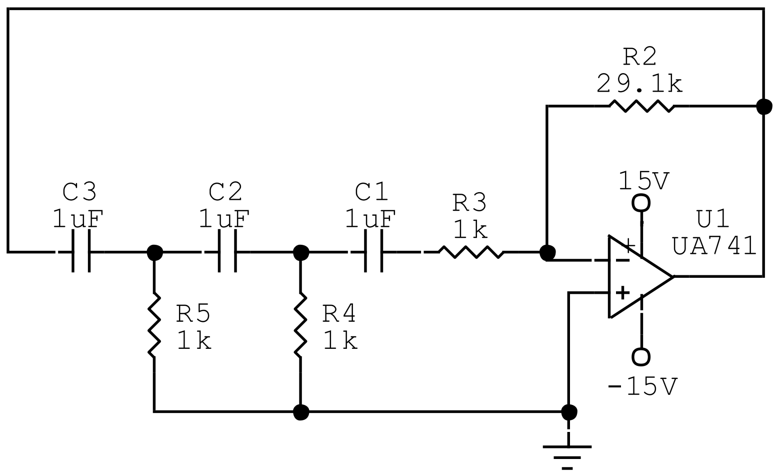




Summary [Part A]:

- From the root locus diagrams, it can be seen that increasing the value for A_M shifts the pole locations to the right. When $A_M = 3$ the poles lie on the imaginary axis. At this point, the output oscillates even with the input grounded as seen by the transient response figure. If A_M is increased further (by increasing R_2 and decreasing R_1) eventually the pole locations with lie on the right hand side of the S plane. This will make the system unstable. It can also be seen that increasing the gain moves you along the root locus diagram. Meaning a high enough gain would eventually result in the poles lying entirely on the real axis and having no imaginary component.

Part B



Phase shift Oscillator

	R	C	Frequency
Original	1kΩ	1uF	65.4Hz
Double	2kΩ	2uF	261.6Hz
Half	0.5kΩ	0.5uF	16.35

Oscillating frequencies for different R/C values

Oscillating frequency calculation:

The oscillating frequencies can be calculated from the following equation, where N is the number of RC stages (ie: 3)

$$f = \frac{1}{2\pi RC\sqrt{2N}}$$

Original values:

$$f = \frac{1}{2\pi(1k)(1uF)\sqrt{6}}$$

$$f = 64.975Hz$$

Double values:

$$f = \frac{1}{2\pi(2k)(2uF)\sqrt{6}}$$

$$f = 16.244Hz$$

Half values:

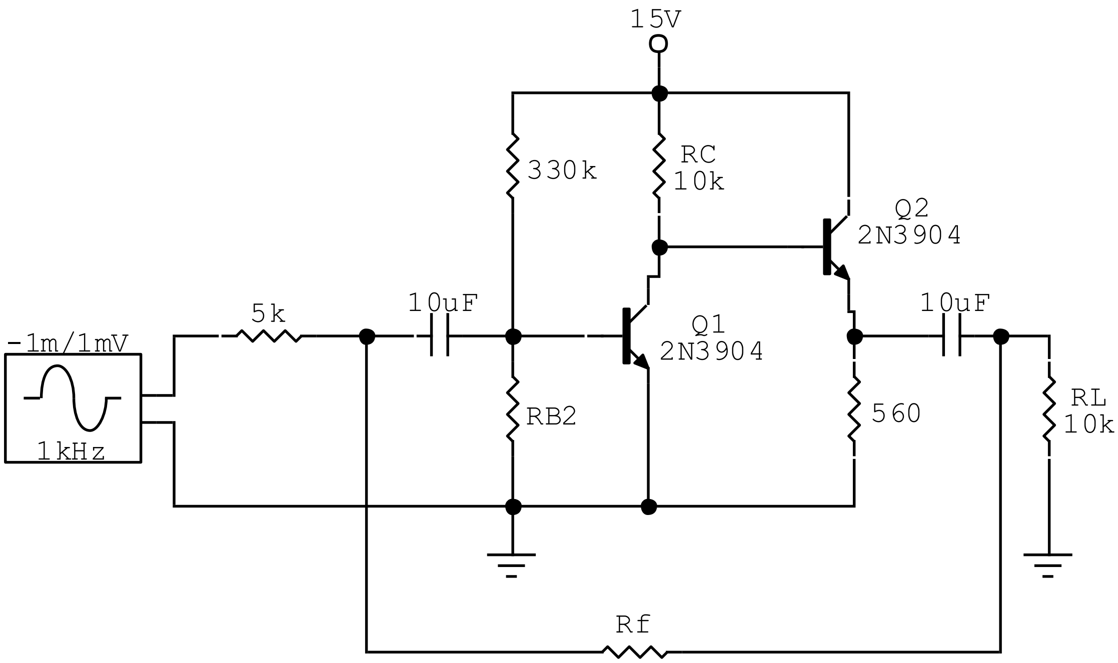
$$f = \frac{1}{2\pi(0.5k)(0.5uF)\sqrt{6}}$$

$$f = 259.899Hz$$

Summary [Part B]:

- The RC oscillator works by combining an amplifier and an RC network to generate oscillations based on the phase shifts between the stages. This specific oscillator had three RC stages as seen in the frequency calcualtion where N was equal to 3. In this circuit, the RC stages also act as an attenuator and generate a total attenuation across its three stages. The gain of the amplifier must be large enough to overcome these losses and 29R is the size where it will match and cancel the RC stage losses such that there is no output oscillation. However, if this resistor is made slightly larger (ie: 29.1k) then the gain is large enough to overcome these losses and we see an output oscillation from the circuit. There is some discrepency between the calculated and measured values which is as expected. However, these differences are quite small and are acceptable given the approximation of measurements.

Part C



Feedback Circuit

By experimenting with different values for R_{B2} , it is found that the largest open loop gain at 1kHz occurs when R_{B2} is approximately 20kΩ

1.

DC Bias Values:

	V_B	V_C	V_E	I_B	I_C	I_E
Q1	0.654V	1.9V	0V	10.77uA	1.295mA	1.305mA
Q2	1.9V	15V	1.235V	15.39uA	2.19mA	2.205mA

D.C. Operating Point

The parameter g_m, r_π, and h_{fe} can be obtained using the following equations

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{h_{fe}}{g_m}$$

$$h_{fe} = \frac{I_C}{I_B}$$

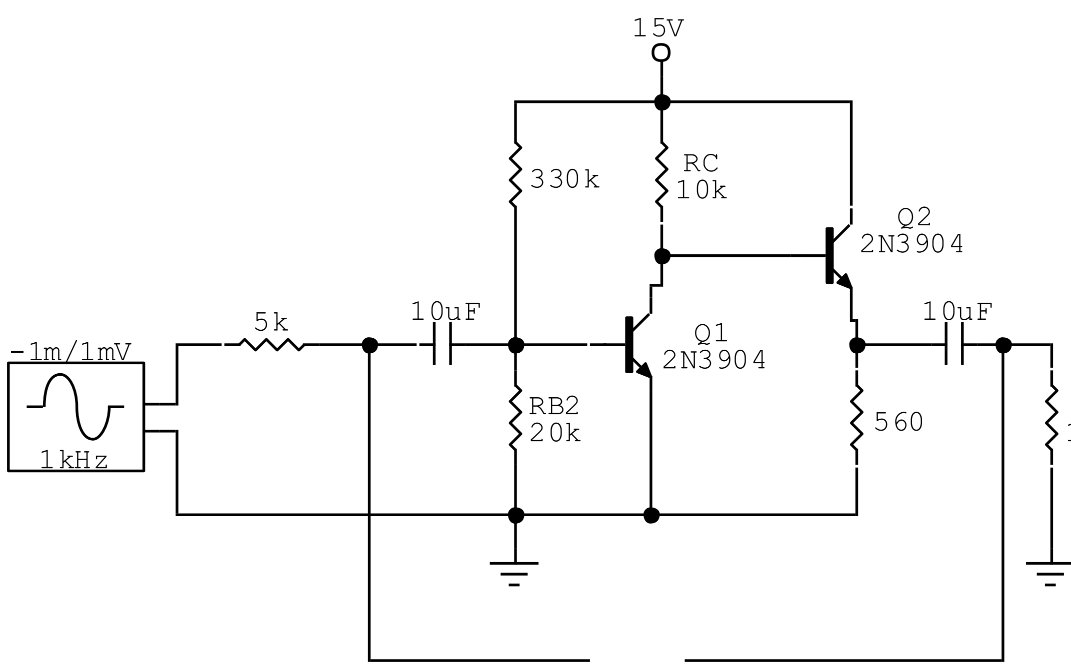
Therefore, for the respective transistors the parameters are as follows

	$\boldsymbol{g_m}$	$\boldsymbol{r_\pi}$	$\boldsymbol{h_{fe}}$
Q1	51.8m℧	2.321kΩ	120.2
Q2	87.6m℧	1.624kΩ	142.3

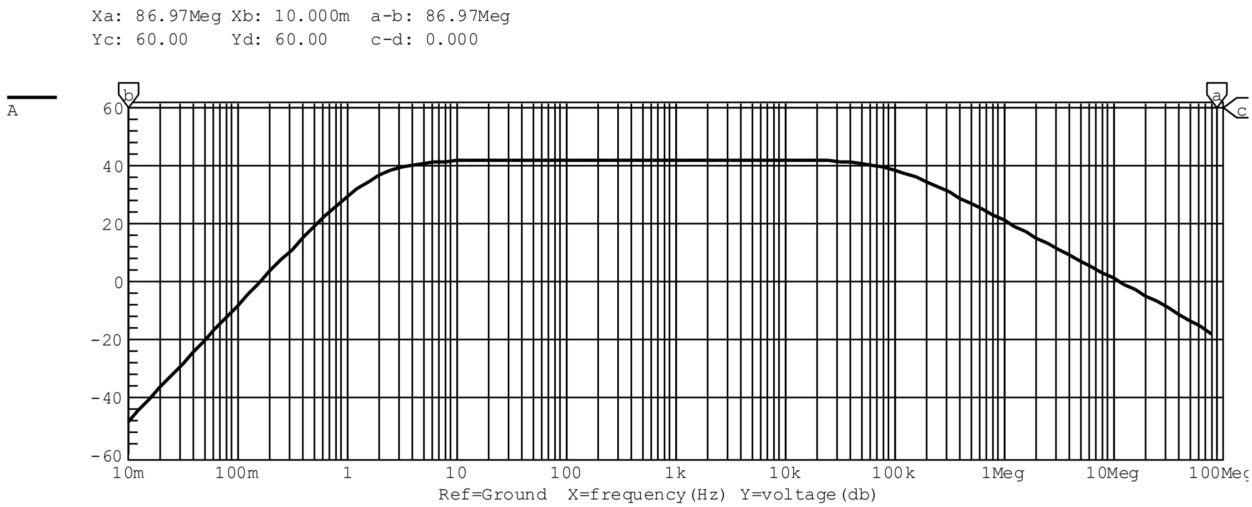
Transistor Parameters

2.

Measured open-loop frequency response:



Feedback Circuit with $\boldsymbol{R_f} = \infty$



Magnitude Plot

From this plot the low and high 3dB points are measured to be

$$\omega_{L3dB} = 2.955Hz$$

$$\omega_{H3dB} = 89.33kHz$$

And the mid band gain is

$$A = 42.12dB$$

$$= -127.64\frac{V}{V}$$

The input and output resistances of this amplifier can be found by applying a test source to the input and output nodes

Input Impedance:

$$R_i = \frac{V_{TEST}}{I_{TEST}}$$

$$= \frac{244.487uV}{94.36nA}$$

$$= 2.591k\Omega$$

Output Impedance:

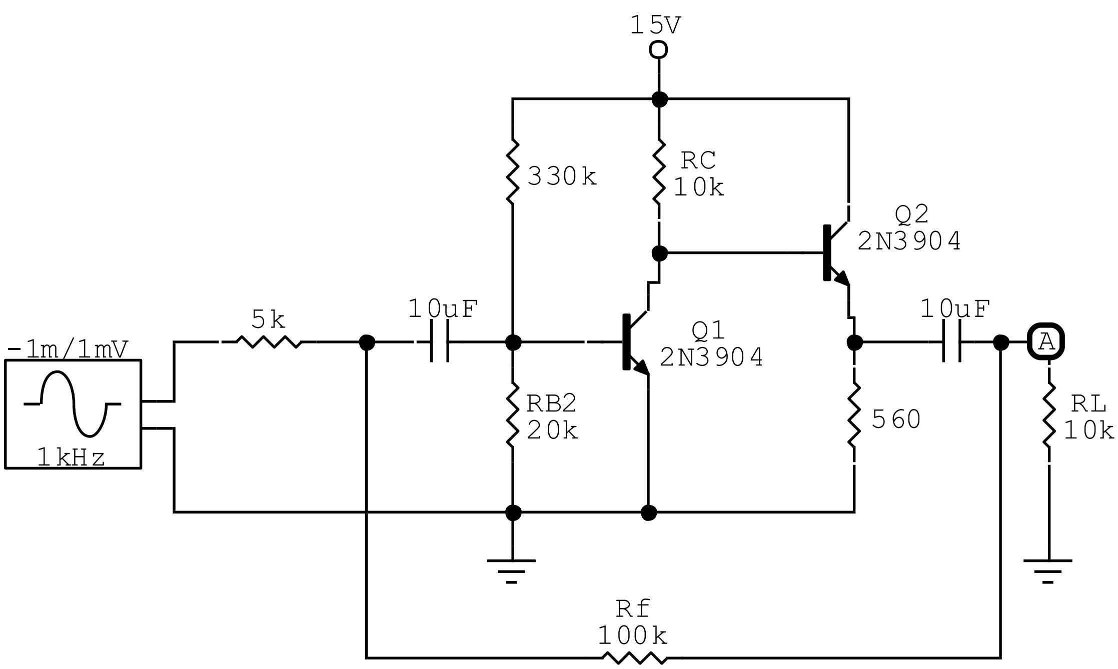
$$R_o = \frac{V_{TEST}}{I_{TEST}}$$

$$= \frac{0.711mV}{11.24nA}$$

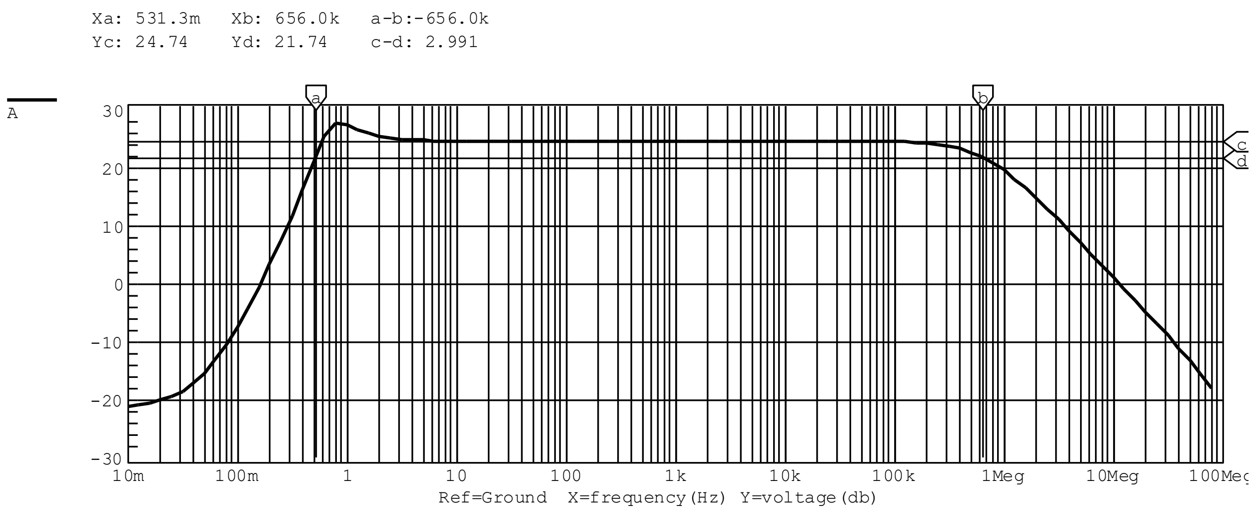
$$= 63.256\Omega$$

Measured closed-loop frequency response:

Now, changing R_f to be 100kΩ



Feedback Circuit with $R_f = 100k\Omega$



Magnitude Plot

From this plot the low and high 3dB points are measured to be

$$\omega_{L3dBf} = 0.531mHz$$

$$\omega_{H3dBf} = 656kHz$$

And the mid band gain is (negative because its an inverting amplifier)

$$A_f = 24.74dB$$

$$= -17.26\frac{V}{V}$$

The input and output resistances of this amplifier can be found by applying a test source to the input and output nodes

Input Impedance:

$$R_{if} = \frac{V_{TEST}}{I_{TEST}}$$

$$= \frac{22.85uV}{95.16nA}$$

$$= 240.12\Omega$$

Output Impedance:

$$R_{of} = \frac{V_{TEST}}{I_{TEST}}$$

$$= \frac{97.35nV}{11.36nA}$$

$$= 8.57\Omega$$

Calculated closed-loop frequency response:

The closed-loop gain can be calculated using the information obtained in the open-loop analysis

$$\beta = \frac{-1}{R_f}$$

$$= \frac{-1}{100k\Omega}$$

$$-10u\mho$$

The open-loop gain A is given by

$$A = R_S \times A_{open}$$

$$= 5k\Omega \times -127.64\frac{V}{V}$$

$$-638.2k\frac{V}{A}$$

Then, the feedback equation can be used to calculate the closed-loop gain

$$A_f = \frac{A}{1 + A\beta}$$

$$= \frac{638.2}{1 + 638.2k \times 10u}$$

$$= 86.45k\frac{V}{A}$$

Dividing by the input voltage, the gain can be obtained in volts/volt

(negative because its an inverting amplifier)

$$\frac{86.45k\frac{V}{A}}{5k\Omega}$$

$$A_f = -17.29\frac{V}{V}$$

Next, the low and high 3dB points can be calculated

$$\omega_{L3dBf} = \frac{\omega_{L3dB}}{1 + A\beta}$$

$$\omega_{L3dBf} = \frac{2.955}{1 + 638.2k \times 10u}$$

$$\omega_{L3dBf} = 0.40Hz$$

$$\omega_{H3dBf} = \omega_{H3dB} \times (1 + A\beta)$$

$$\omega_{H3dBf} = 89.33k \times (1 + 638.2k \times 10u)$$

$$\omega_{H3dBf} = 659.43kHz$$

And finally, the input and output impedances can be calculated

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{if} = \frac{2.591k}{1 + 638.2k \times 10u}$$

$$R_{if} = 351\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

$$R_{of} = \frac{63.256}{1 + 638.2k \times 10u}$$

$$R_{of} = 8.57\Omega$$

Comparison [Measured vs Calculated Values]:

	$\boldsymbol{A_f}$	$\boldsymbol{\omega_{L3dBf}}$	$\boldsymbol{\omega_{H3dBf}}$	$\boldsymbol{R_{if}}$	$\boldsymbol{R_{of}}$
Measured	$-17.26\frac{V}{V}$	$0.531mHz$	$656kHz$	240.12Ω	8.57Ω
Calculated	$-17.29\frac{V}{V}$	$0.40Hz$	$659.43kHz$	351Ω	8.57Ω

Discussion [Measured vs Calculated Values]:

- The closed-loop gain, high 3dB point, and output impedance all correspond highly with one another and are nearly identical in some cases. The input resistance bears some inaccuracy with its calculated value; however this is still a very good approximation. The biggest distinction among values is between the calculated and measured values for the low 3dB point. This is attributed most likely to the spike in the magnitude plot near the low 3dB point. It is useful to notice that this spike did not occur for the open-loop response when $R_f = \infty$ and the effect would be greater if the value for R_f was reduced. Overall, the calculated and measured values match up quite nicely which gives further confidence to the tools and methods learned in class.

3.

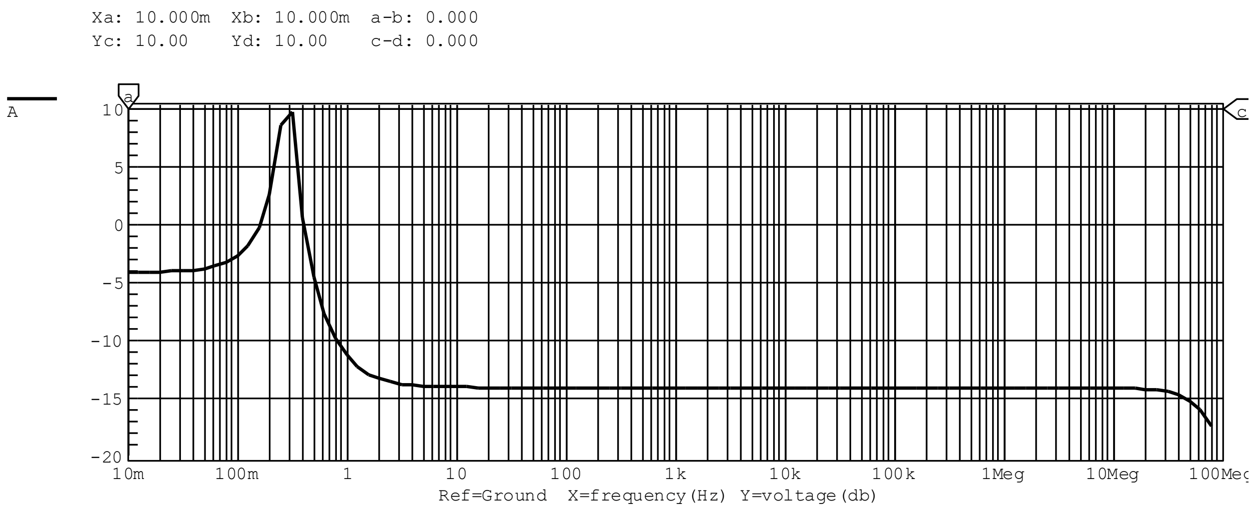
For the different R_f values, β can be easily calculated as being

$$\beta = -\frac{1}{R_f}$$

β can be measured from the plots by using the equation below and A which was calculated in part 2 to be $-638.2k\,\frac{V}{A}$

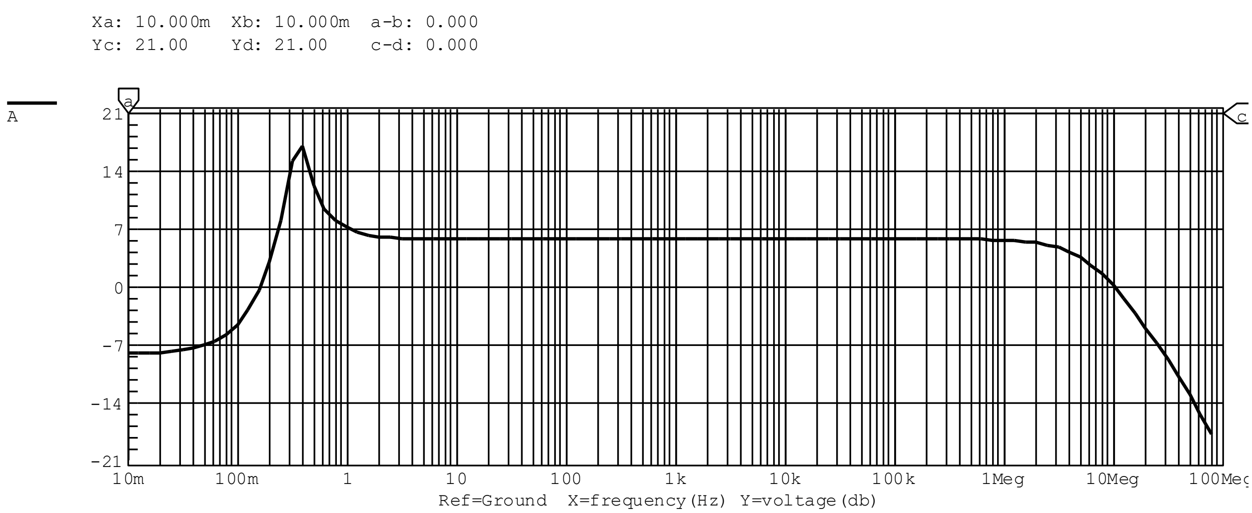
$$A_f = \frac{A}{1 + A\beta}$$

Magnitude Plots:



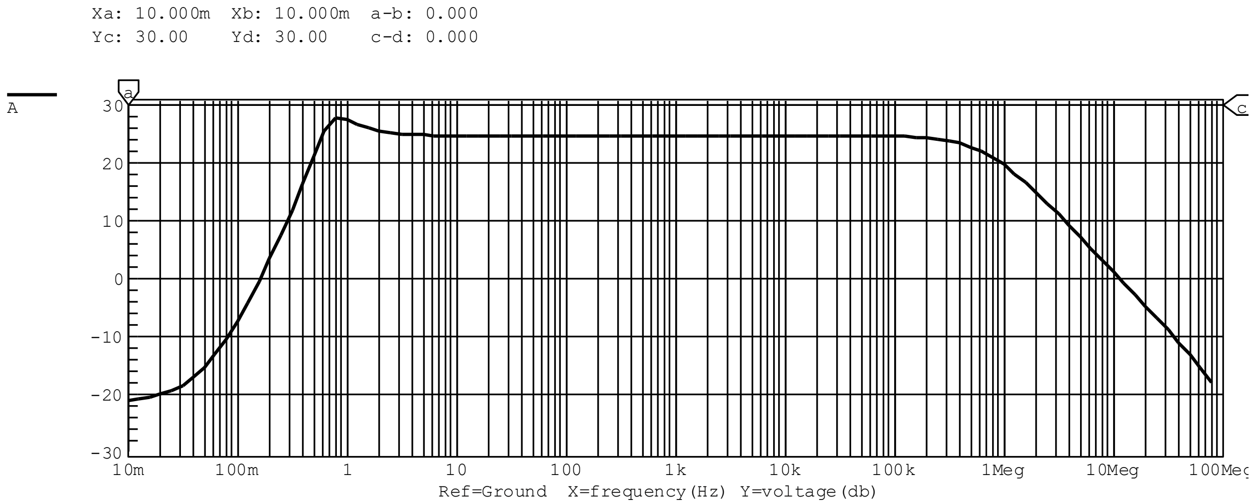
$$\boldsymbol{R_f = 1k}\Omega$$

$$A_f = 13.97dB$$



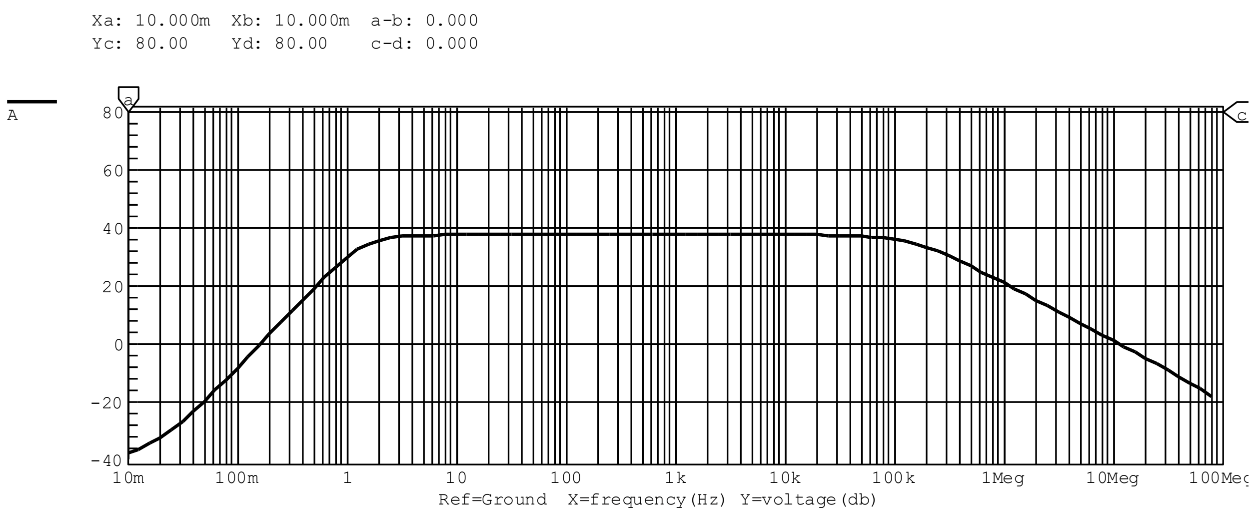
$$\boldsymbol{R_f = 10k}\Omega$$

$$A_f = 5.921dB$$



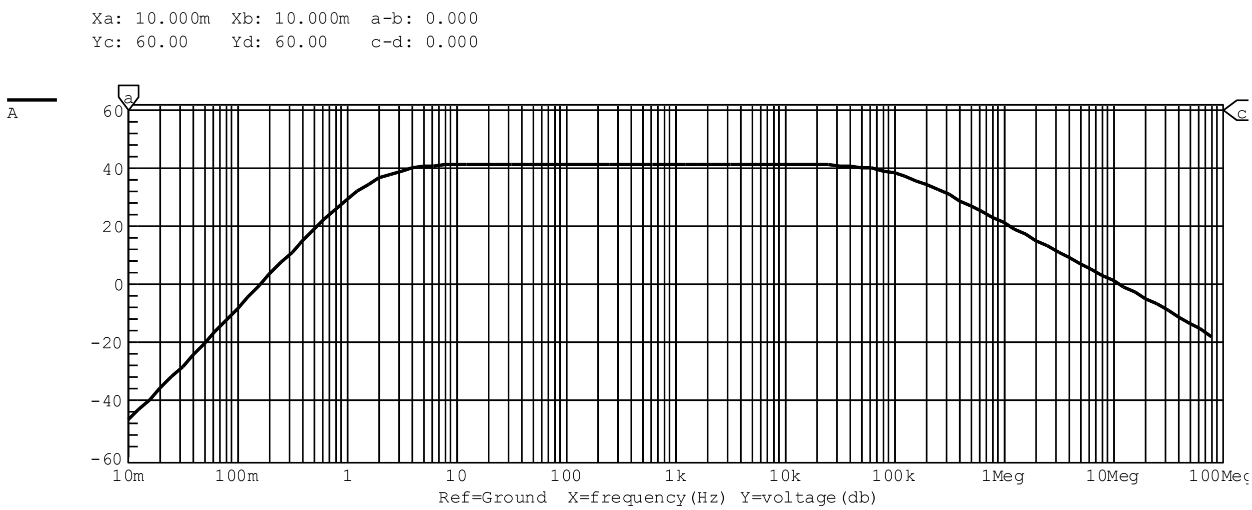
$$\boldsymbol{R_f = 100k}\Omega$$

$$A_f = 24.83dB$$



$$\boldsymbol{R_f = 1M}\Omega$$

$$A_f = 38.76dB$$



$$\boldsymbol{R_f = 10M\Omega}$$

$$A_f = 41.13dB$$

Using the gain obtained from the plots and A from part 2, the measured value of β is calculated and displayed in the table below for the corresponding R_f values and is compared to the calculated value from the equation $\beta = -\frac{1}{R_f}$

R_f	**1k**	**10k**	**100k**	**1M**	**10M**
Measured (β)	-1.00×10^{-3}	-1.02×10^{-4}	-1.00×10^{-5}	-0.99×10^{-6}	-0.93×10^{-7}
Calculated(β)	-1.00×10^{-3}	-1.00×10^{-4}	-1.00×10^{-5}	-1.00×10^{-6}	-1.00×10^{-7}

Discussion [Measured vs Calculated Values]:

- It can be seen from this analysis that the measured and calculated values match up quite closely. This is reaffirming to the tools/methods use to obtain these results. It can also be noticed from the plots that higher values for R_f provided a much smoother magnitude plot with no significant spike near the low 3dB point.

4.

The input and output resistances of this amplifier can be found by applying a test source to the input and output nodes

Input Impedance:

$$R_{if} = \frac{V_{TEST}}{I_{TEST}}$$

Output Impedance:

$$R_{of} = \frac{V_{TEST}}{I_{TEST}}$$

I did this for the different values of R_f as shown in the table below

$\boldsymbol{R_f}$	$\boldsymbol{R_{if}}$	$\boldsymbol{R_{of}}$
10kΩ	25.37Ω	1.12Ω
100kΩ	243.65Ω	8.57Ω
1MΩ	1.28kΩ	36.31Ω

The amount of feedback can then be estimated from the following equations and values for R_i, R_o, and A coming from the previous sections

$$R_{if} = \frac{R_i}{1 + A\beta}$$

$$R_{of} = \frac{R_o}{1 + A\beta}$$

These can then be compared with the feedbacks obtained in part 3 using gain instead of impedances

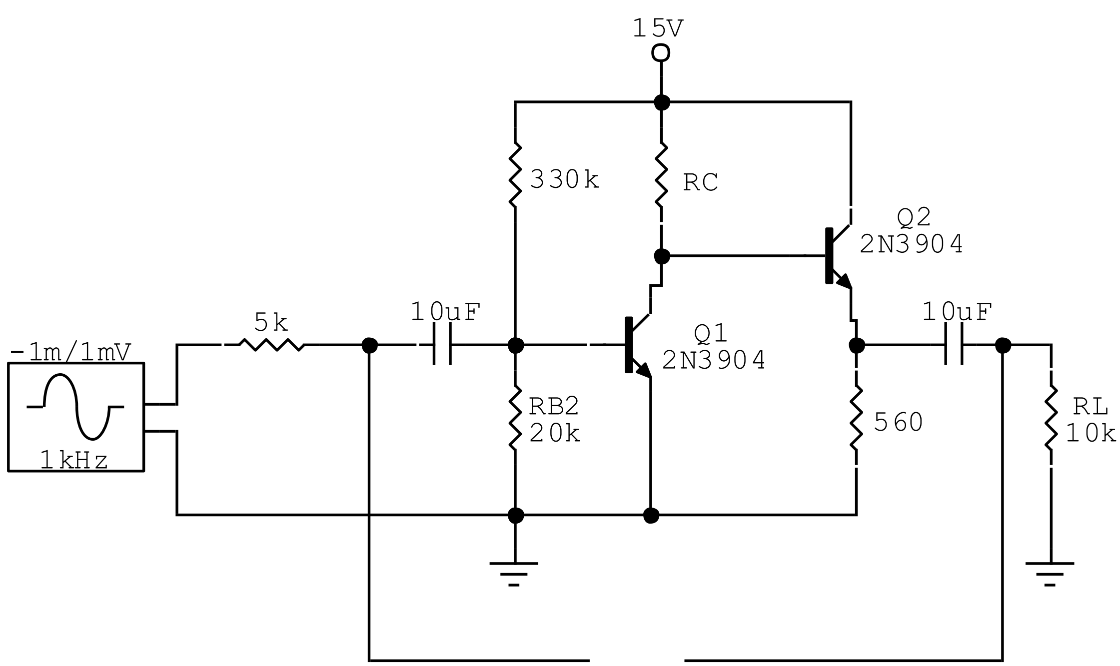
R_f	1k	10k	100k
β (from R_{if})	-1.56×10^{-4}	-1.54×10^{-5}	-1.54×10^{-6}
β (from R_{of})	-8.57×10^{-5}	-9.93×10^{-6}	-1.31×10^{-5}
β (from Part 3)	-1.00×10^{-3}	-1.02×10^{-4}	-1.00×10^{-5}
β (from $\beta = -\frac{1}{R_f}$)	-1.00×10^{-3}	-1.00×10^{-4}	-1.00×10^{-5}

Discussion [Estimating Feedback using Gain vs Impedance]:

- It is immediately clear in this analysis that gain was a much better predicter of the amount of feedback when compared to using either the input or output impedances. Within the impedance calculations themselves, it seems that using R_{if} was at the very least more consistent and probably a better overall predicter of feedback. However, it was still much less accurate then using the gain from part 3 as shown in the table above.

5.

The "desensitivity factor" is $1 + A\beta$. First, I will look at the open-loop desensitivity factor with $R_f = \infty$



Feedback Circuit with $R_f = \infty$ and variable R_C

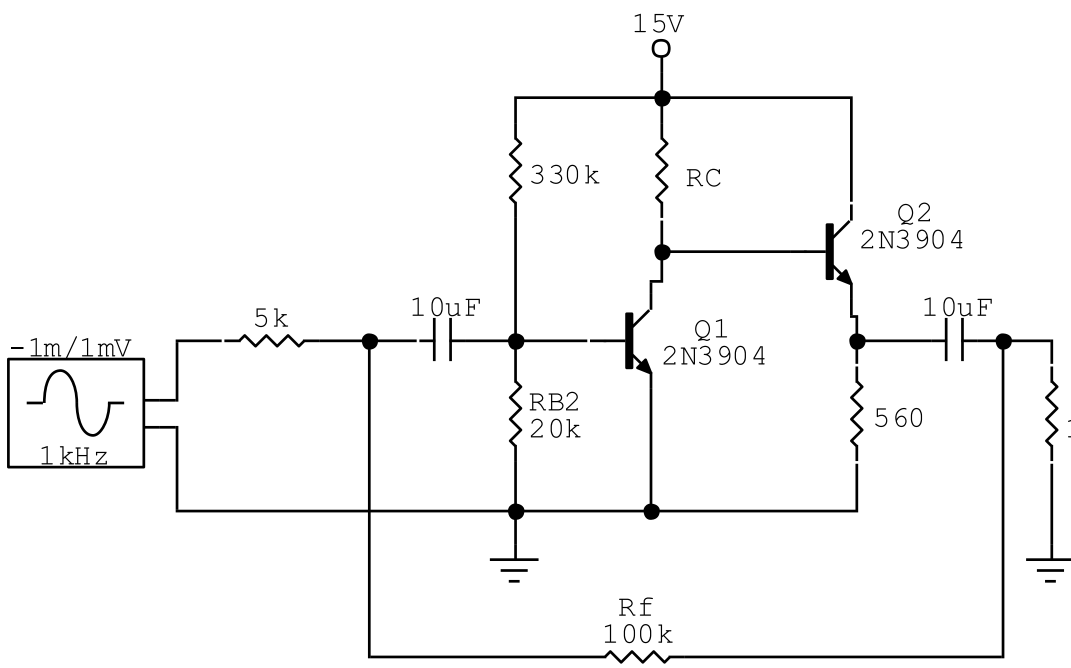
R_C	**9.9kΩ**	**10kΩ**	**10.1kΩ**
A	$-126.92\frac{V}{V}$	$-127.64\frac{V}{V}$	$-128.35\frac{V}{V}$

The desensitivity factor is

$$1 + A\beta$$
$$= 1 + 0$$
$$= 1$$

Which is as expected, because $R_f = \infty$, the circuit is open-loop and there is no feedback. Changing R_C does influence the gain slightly but the term is brought to zero by the large R_f

Now, I will introduce feedback by setting $R_f = 100k\Omega$



Feedback Circuit with $R_f = 100k\Omega$ and variable R_C

R_C	**9.9kΩ**	**10kΩ**	**10.1kΩ**
A_f	$-17.26\frac{V}{V}$	$-17.28\frac{V}{V}$	$-17.29\frac{V}{V}$

The desensitivity factor can now be calculated for each of the R_C values using $1 + A\beta$. I will use D_f to denote the desensitivity factor

$\boldsymbol{R_C}$	**9.9kΩ**	**10kΩ**	**10.1kΩ**
$\boldsymbol{D_f}$	8.354	8.361	8.376

Summary [Part C]:

- In this section, feedback was examined by changing the value for the R_f resistor. When R_f was set to ∞, an open-loop system was obtained. In this situation the desensitivity factor was 1, and there was no influence on the gain. When R_f was set to a range of values (1k-1M), feedback now played a role in the overall circuit with an added desensitivity factor which influenced the gain. This amount of feedback was calculated in two different ways. First, gain was used to predict the feedback repsonse. Next, the input and output impedances were used to do the same. As discussed earlier, gain is a far better predicter of the amount of feedback and using the impedances introduced some error. One of the things discovered in this section is that smaller values of R_f created a spike around the low 3dB point that is larger than the gain at midband. Recall that $\beta = \frac{1}{R_f}$, so for small values for R_f we get a very large value for β. This allows this term to dominate the gain equation and create the behaviour seen in the circuits with low R_f values. The circuits that had larger R_f values saw a much smoother transition into the midband.